

Marketmakers versus Matchmakers*

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This paper examines why we have marketmakers (specialists in stock markets, used-car dealers) in some markets and matchmakers (real-estate brokers, employment agencies) in others. Using a bilateral search model, it is shown that when the valuations of the agents are private information, marketmaking might yield higher or lower profits and welfare effects than matchmaking, depending on the efficiency and the cost of search and on the distribution of valuations of the agents. This is in contrast to an earlier result that when the agents' valuations are common knowledge marketmaking yields higher profits and greater welfare effects than matchmaking. *Journal of Economic Literature* Classification Numbers: G0, D8.

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I. INTRODUCTION

We observe two types of middleman in exchange markets. One is a marketmaker who sets an ask price and a bid price at which she sells and buys on her own account. The other is a matchmaker who does not sell or buy, but simply matches two parties (buyers with sellers, firms with workers, etc.).¹ Examples of marketmakers include specialists in stock markets and used-car dealers, while examples of matchmakers include real-

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¹ The literature uses different terms for these two middleman types. While Garman (1976), O'Hara and Oldfield (1986), and Peck (1990) employ the term "marketmakers," Stoll (1985) utilizes the term "dealers," and Niehans (1978) uses the term "qualitative asset transformers." Stoll (1985) and Niehans (1978) also use "brokers" in lieu of "matchmakers."

estate brokers, travel agents, and employment agencies. Both types of middleman play significant roles in the economy. Specialists, for instance, participated in more than 72% of the total volume of trade in NYSE in 1980 (Stoll, 1985, p. 12), and approximately 80% of all sellers of residential houses in the United States between 1978 and 1981 used real-estate brokers (Federal Trade Commission, 1983, p. 8).

An interesting question is Why do we have marketmakers in some exchange markets and matchmakers in others? The question is important because the type of middleman in a market affects welfare. We examine the marketmaker/matchmaker choice from two perspectives:

1. Which type of middleman will emerge in a market if we let the middleman choose between being a marketmaker or a matchmaker (i.e., which type of intermediation service will yield higher expected profits)?
2. Which type of a middleman will emerge in a market if a benign planner appoints someone to provide the intermediation service (i.e., which type of a middleman will maximize welfare)?

These two questions will be analyzed in a bilateral search framework involving two agents (a buyer and a seller, a firm and a worker, a man and a woman, etc.) searching for each other in order to trade. In addition to searching, the agents also have the option of trading through a middleman. The reason for approaching a middleman is that she can potentially reduce two inefficiencies in search economies: the uncertainty that search efforts of the agents may not result in a match and the externalities that exist in the matching process. The middleman reduces the uncertainty and/or internalizes some of the externalities in return for some profits.

In Yavaş (1991b), I show that when the valuations of the agents are common knowledge, marketmaking yields more profits and higher welfare than matchmaking. Here I assume instead that valuations of the agents are private information. It turns out that the information structure is critical. When the valuations are private information, marketmaking might yield higher or lower profits and welfare than matchmaking does, depending on factors that include the efficiency and cost of the search and the distribution of valuations of the agents. If, for example, search is efficient and costless, then the middleman prefers to be a marketmaker. If, on the other hand, search is relatively inefficient and sufficiently costly, matchmaking produces higher profits and welfare.

Recently, there have been a number of studies on the role of middlemen in search economies. Gehrig (1990), Moresi (1990), Rubinstein and Wolinsky (1987), and Yavaş (1990a, 1991a) examine the role of middlemen in search economies with a unique equilibrium, while Bhattacharya and Hagerty (1987) and Yavaş (1990b) examine the role of middlemen in search economies displaying multiple equilibria. The emphasis in Peck

(1990) is on the welfare impact of the price-smoothing role of the middleman. The middleman in all of these studies, except Yavaş (1991a), is a marketmaker. While this literature is tangentially related to this paper, the focus here is quite different.

The rest of the paper is organized as follows. Section IIa develops a bilateral search model with no intermediation. Sections IIb and IIc generalize the model to study the role of marketmakers and matchmakers, respectively. Section IIc compares the two intermediaries in terms of their welfare effects and expected profits. Section III discusses the results of the paper and some possible extensions.

II. THE SEARCH MODEL

a. *No Middleman*

First, we present the basic search model without any middleman. This is a one-period bilateral search model in which two agents search for each other in order to trade an identical commodity. The agents are assumed to be risk neutral and indexed by $i = 1, 2$. Each agent has one unit of the commodity and an endowment of a numeraire good.² An agent searches for the other agent to sell his unit or to buy another unit. The model does not separate agents into two classes, buyers and sellers; instead, at a given price, an agent might become a buyer or a seller depending upon his liquidity needs, information, portfolio considerations, etc.

Let P_i be the valuation of the unit by agent i , $i = 1, 2$. Each agent knows his own valuation but not the valuation of the other agent until they meet. For Agent 1, the valuation of Agent 2 is a random draw from a probability distribution $F(\cdot)$, and for Agent 2 the valuation of Agent 1 comes from a probability distribution $G(\cdot)$. $F(\cdot)$ is continuous and strictly increasing on the interval $[\underline{P}_2, \bar{P}_2] \in \mathbb{R}$ with $F(\underline{P}_2) = 0$ and $F(\bar{P}_2) = 1$, and $G(\cdot)$ is continuous and strictly increasing on the interval $[\underline{P}_1, \bar{P}_1] \in \mathbb{R}$ with $G(\underline{P}_1) = 0$ and $G(\bar{P}_1) = 1$. Of interest is the rational expectations equilibrium where distributions $F(\cdot)$ and $G(\cdot)$ are known to both sides.³ Since the valuation of one side is a random draw for the other side, the surplus received by an agent varies depending on luck in sampling. When a match is obtained, the valuations of both sides are revealed and the price is

² In order to avoid the additional questions associated with having money in the economy, this endowment has not been labeled as money.

³ Rothschild (1974) relaxes the very common assumption that searchers know the probability distribution from which they are searching. He concludes that the qualitative properties of optimal search from unknown distributions are similar to those that are from known distributions.

negotiated instantaneously. It is assumed that the negotiated price splits the surplus between the two agents.⁴ Hence, if $P_1 > P_2$, Agent 2 sells his unit to Agent 1 at a price of $P_2 + \frac{1}{2}(P_1 - P_2)$, and if $P_2 > P_1$, Agent 2 buys a unit from Agent 1 at a price of $P_1 + \frac{1}{2}(P_2 - P_1)$. In either case, the surplus $|P_1 - P_2|$ is divided equally between the two agents. For the current model, this is the bargaining solution proposed by Nash (1950), Kalai and Smorodinsky (1975), and Rubinstein (1982).

Each agent chooses a search intensity to maximize his expected return from search. The choice of the search intensity affects the probability and the cost of meeting the other agent. Let $A \in [0, M]$ be the search intensity of Agent 1 and let $B \in [0, M]$ be the search intensity of Agent 2, where M is finite and bounds the agents' search intensity levels. The probability that the two agents meet is given by the matching technology $0 \leq \theta(A, B) = A^m + B^m \leq 1$, $1 > m > 0$. It is assumed that $M \leq (\frac{1}{2})^{1/m}$ so that $\theta(M, M) \leq 1$. The cost of search is given by $C(A) = A^n$ for Agent 1 and $C(B) = B^n$ for Agent 2, $n > 1$. The functions $\theta(A, B)$, $C(A)$, and $C(B)$ are common knowledge. These functional forms have been chosen for two reasons. One is that they allow a simple and clear presentation of the main ideas of the paper; the existence of a continuum of agent types makes the analysis difficult when there is strategic interaction among the agents' search activities. The second reason is that the parameters m and n permit an explicit expression of the efficiency and cost of the search. As m increases, the probability of a match gets smaller (note that A and B cannot exceed M , which cannot exceed $(\frac{1}{2})^{1/m}$), and hence the search becomes less efficient. Meanwhile, an increase in n makes the search less costly. Furthermore, it is shown in Sections IIb and IIc that the choice of

⁴ The choice of the bargaining solution is not critical; changing the bargaining solution affects the share of the surplus that each agent gets, but not the qualitative results of the paper. We can also relax the assumption that the valuations of the agents are revealed when they meet. This, as shown in Chatterjee and Samuelson (1983), creates the possibility that the optimal bargaining strategies of the agents *may* result in the breakdown of negotiations altogether, despite the fact that there are mutual benefits from trade. This will affect the expected gains of the agents from search in our model, but it will have no qualitative impact on the results of the paper as long as each agent has a positive probability of obtaining a positive surplus from the bargaining game (otherwise he will have no incentive to search). An alternative way of modeling the bargaining game here is to assume that one of the agents makes repeated offers to the other agent. Ausubel and Denekere (1989) study such a bargaining game. They prove that even if the agent making the offers has monopoly power, there exist subgame perfect equilibria where he does not extract all the surplus. Bester (1991) goes a step further and compares the bargaining game where the sellers make a "take it or leave it" offer to the buyers with the bargaining game where the two sides engage in bilateral bargaining and studies the conditions under which one institution will prevail over the other in the market. This paper does not model the bargaining process, and, as pointed out above, the qualitative results of the paper are robust to the choice of bargaining game as long as it yields a positive expected surplus from search for each agent.

this matching technology yields an interesting result, namely that the introduction of the middleman into this search economy induces strategic interactions between the search intensities of the agents.

Since P_2 is a random draw for Agent 1, the expected surplus of Agent 1 of type P_1 associated with a match becomes

$$R_1(P_1) = \int_{\underline{P}_2}^{\bar{P}_2} \frac{1}{2} |P_1 - \tilde{P}_2| dF(\tilde{P}_2), \quad (1)$$

where “ $\sim$ ” is used to denote a random variable.

Since Agent 1 does not know the valuation of Agent 2, he uses his conjecture about the search intensity function of Agent 2, $B^\circ(\cdot)$. He anticipates that Agent 2 will choose $B^\circ(P_2)$ if he has valuation P_2 ; however, not knowing P_2 , Agent 1 must compute his gain from search in expectations. Given his conjecture, Agent 1 of type P_1 solves

$$\max_{M \geq A \geq 0} V_1(A, P_1, B^\circ(\cdot)) = \int_{\underline{P}_2}^{\bar{P}_2} [A^m + (B^\circ(\tilde{P}_2))^m]^{\frac{1}{2}} |P_1 - \tilde{P}_2| dF(\tilde{P}_2) - A^n. \quad (2)$$

Agent 2's problem is symmetric to that of Agent 1,

$$\max_{M \geq B \geq 0} V_2(B, P_2, A^\circ(\cdot)) = \int_{\underline{P}_1}^{\bar{P}_1} (B^m + (A^\circ(\tilde{P}_1))^m)^{\frac{1}{2}} |\tilde{P}_1 - P_2| dG(\tilde{P}_1) - B^n \quad (3)$$

with

$$R_2(P_2) = \int_{\underline{P}_1}^{\bar{P}_1} \frac{1}{2} |\tilde{P}_1 - P_2| dG(\tilde{P}_1) \quad (4)$$

as his expected surplus from a match.

Note that the search intensities of the agents are strategically independent. That is, a change in the search intensity of one agent does not affect the search intensity choice of the other agent. Given this, the equilibrium will consist of a pair of dominant strategies, one for each agent. These are given by the following search intensity functions

$$A^*(P_1) = [mR_1(P_1)/n]^{1/(n-m)} \quad (5)$$

and

$$B^*(P_2) = [mR_2(P_2)/n]^{1/(n-m)}. \quad (6)$$

The equilibrium in this search economy involves two sources of inefficiencies. One is the uncertainty that the search efforts of the agents may not result in a match. The other is the positive externalities that exist in the matching process. These externalities result from the fact that an increase in the search intensity of one agent increases the probability of a match, hence the payoffs to the other agent. Because each agent receives a portion of the surplus from a match, these externalities result in search intensities less than the amounts that maximize the agents' joint payoff. These two inefficiencies could give rise to an institution that could reduce the uncertainty and internalize some of the externalities in return for some profits. As a next step, we introduce intermediation as such an institution into this search economy.

Two different types of middlemen will be examined here. One is a marketmaker who sets an ask price and a bid price at which she sells and buys on her own account. The other is a matchmaker who does not buy or sell, but simply matches the two agents. Both types of intermediary are assumed to be risk neutral and endowed with monopoly power.

b. *Marketmaker*

A marketmaker sets an ask price, α , at which she sells, and a bid price, β , at which she buys. α and β maximize her expected profits. She is assumed to have two units of endowment, which enable her to meet all potential buy and sell orders at α and β (both agents might choose to buy a unit from the marketmaker at α or sell a unit to her at β); i.e., she is capable of providing the *service of immediacy*. This is the kind of service provided by specialists in stock markets and by intermediaries in agricultural markets. The marketmaker's valuation of the unit is given by I . She knows the cost functions and the distributions $F(\cdot)$ and $G(\cdot)$, but not the actual valuations of the agents.

The marketmaker posts the ask and bid prices at the beginning of the period, and agents are assumed to know these prices before they choose their search intensities. Agents can trade with the marketmaker instantaneously and at no cost. At the beginning of the period, an agent has two strategies: he can either trade directly with the marketmaker, or he can search for the other agent. If he does not meet the other agent, he can still go back and trade with the marketmaker. However, if an agent trades with the marketmaker after having already searched, his surplus is assumed to be discounted by a constant, δ , $0 \leq \delta \leq 1$. Suppose, for example, that Agent 1's valuation is given by $P_1 < \beta$. If he sells his unit directly to the marketmaker, he gets $\beta - P_1$. If he chooses to search, he has a chance to find Agent 2 and receive a surplus greater than $\beta - P_1$. It is possible that search does not result in a match ($\theta(\cdot) < 1$), in which case he can go back to the marketmaker and obtain $\delta(\beta - P_1)$.

Because the agents can still trade with the marketmaker after having searched, the surplus that they can receive from trading with the marketmaker constitutes a threat point in their bargaining process with each other. The bargaining rule divides the surplus in the following way. When the two agents meet, each party first receives the surplus attainable from trading with the marketmaker. This surplus is $\delta(P_i - \alpha)$ if $P_i > \alpha$, $\delta(\beta - P_i)$ if $\beta > P_i$, and 0 if $\alpha > P_i > \beta$, since agent i can buy a unit from the marketmaker if $P_i > \alpha$ or sell his unit to the marketmaker if $P_i < \beta$, but cannot trade with the marketmaker if $\alpha > P_i > \beta$. The surplus over and above that attainable from trading with the marketmaker is then divided equally between the two agents. If, for example, $P_2 > \alpha$, $P_1 < \beta$, and $P_2 > P_1$, then Agent 1 gets $\delta(\beta - P_1) + \frac{1}{2}[(P_2 - P_1) - \delta(\beta - P_1) - \delta(P_2 - \alpha)]$ and Agent 2 receives $\delta(P_2 - \alpha) + \frac{1}{2}[(P_2 - P_1) - \delta(\beta - P_1) - \delta(P_2 - \alpha)]$. The bargaining solutions for different combinations of P_1 , P_2 , α , and β are given in the Appendix. As shown there, the two parties will trade with each other iff the difference between their valuations is large enough so that $|P_2 - P_1| > \delta[\max\{0, P_1 - \alpha, \beta - P_1\} + \max\{0, P_2 - \alpha, \beta - P_2\}]$.

The fact that the agents have the option of selling to the marketmaker at β or buying from her at α affects their reservation prices. The result, as proved in Proposition 1 below, will be a decrease in the price dispersion. This result asserts that, like the specialist in the stock market, the marketmaker reduces price volatility.⁵

PROPOSITION 1. *There is a smaller price dispersion under the marketmaker.*

Proof. Suppose that the two agents form a match and that their valuations are $P_i > P_j$, $i = 1, 2$, $j = 1, 2$, and $i \neq j$. If there is not a marketmaker, an agent's reservation price is simply his valuation, P_i or P_j . Agent j is willing to sell his unit for any price above P_j , and agent i is willing to buy a unit at any price below P_i . If there is a marketmaker, however, agent j will never sell his unit at a price below $P_j + \delta \max\{0, P_j - \alpha, \beta - P_j\}$, and agent i will never buy a unit at a price above $P_i - \delta \max\{0, P_i - \alpha, \beta - P_i\}$. In other words, the marketmaker increases the reservation price of the potential seller ($P_j + \delta \max\{0, P_j - \alpha, \beta - P_j\} \geq P_j$) and decreases the reservation price of the potential buyer ($P_i - \delta \max\{0, P_i - \alpha, \beta - P_i\} \leq P_i$), thereby narrowing the range of prices at which transactions can take place. ■

⁵ Peck (1990) shows that with uncertainty about the demand and supply schedules, this price-smoothing role of the marketmaker might lead to an allocation that Pareto dominates the allocation resulting from a Walrasian auctioneer. Chan (1983) also derives the result that intermediation has a positive welfare effect. His result, however, is due to the middleman's ability to utilize the economies of scale in information acquisition.

Note that the transaction price in the presence of a marketmaker (the average of the agents' reservation prices, $\frac{1}{2}[P_j + \delta \max\{0, P_j - \alpha, \beta - P_j\} + P_i - \delta \max\{0, P_i - \alpha, \beta - P_i\}]$) can be higher or lower than the transaction price in the absence of the marketmaker ($\frac{1}{2}(P_i + P_j)$), depending on the realizations of P_i and P_j vis-à-vis the ask and bid prices of the marketmaker. The marketmaker then establishes an upper bound, $P_i - \delta \max\{0, P_i - \alpha, \beta - P_i\}$, and a lower bound, $P_j + \delta \max\{0, P_j - \alpha, \beta - P_j\}$, on the transaction price. In the case for which $\delta = 1$, $P_i > \alpha$, and $\beta > P_j$, for example, the upper bound becomes α and the lower bound becomes β .

If an agent chooses not to search, then he will not wait to be contacted by the other agent, but instead will go directly to the marketmaker. This is because the marginal cost of zero search is zero, whereas the marginal benefit of zero search is infinite. As a result, if an agent prefers waiting to be contacted over going directly to the marketmaker, he will be better off searching for the other agent. This, in turn, implies that if Agent 1 searches, he will meet only those Agent 2 types who search. Similarly, if Agent 2 searches, he will contact only those Agent 1 types who search.⁶ Therefore, it will be incorrect to use the distributions $F(\cdot)$ and $G(\cdot)$ under the marketmaker, because using them means Agent 1 (Agent 2) can meet any Agent 2 (Agent 1) types if he searches. For that reason the distributions $F(\cdot)$ and $G(\cdot)$ will be replaced with $F^\circ(\cdot)$ and $G^\circ(\cdot)$, where

$$F^\circ(\tilde{P}_2) = \begin{cases} F(\tilde{P}_2) & \text{if } B'(\tilde{P}_2) > 0 \\ 0 & \text{if } B'(\tilde{P}_2) = 0 \end{cases} \quad \text{and}$$

$$G^\circ(\tilde{P}_1) = \begin{cases} G(\tilde{P}_1) & \text{if } A'(\tilde{P}_1) > 0 \\ 0 & \text{if } A'(\tilde{P}_1) = 0, \end{cases}$$

with $B'(\tilde{P}_2)$ as Agent 1's conjecture about the search intensity of Agent 2 of type $\tilde{P}_2$ and $A'(\tilde{P}_1)$ as Agent 2's conjecture about the search intensity of Agent 1 of type $\tilde{P}_1$.

Given that P_2 is a random draw for Agent 1, the net expected return from a match (net of the surplus that the agent can get from trading with the marketmaker) for Agent 1 of type P_1 and Agent 2 of type P_2 , respectively, are given by

⁶ It has been proved in Yavaş (1990a), Gehrig (1990), and Moresi (1990) that agent types with higher gains from trade (those with very high or very low valuations) will choose to trade directly with the marketmaker, while agent types with lower gains from trade (agents with middle valuations) will choose to search. There are two reasons for this result. First, there is an uncertainty that search will fail to yield a match, whereas the gains from trading with the marketmaker are certain. Second, even if there is a match, an agent gets half of the increase in the surplus, whereas he gets all of the increase in the surplus if he trades with the marketmaker.

$$R'_1(P_1, \alpha, \beta) = \int_{P_2}^{\bar{P}_2} \frac{1}{2} \max\{0, |P_1 - \bar{P}_2| - \delta \max\{0, P_1 - \alpha, \beta - P_1\} - \delta \max\{0, \bar{P}_2 - \alpha, \beta - \bar{P}_2\}\} dF^\circ(\bar{P}_2) \quad (7)$$

and

$$R'_2(P_2, \alpha, \beta) = \int_{P_1}^{\bar{P}_1} \frac{1}{2} \max\{0, |\bar{P}_1 - P_2| - \delta \max\{0, \bar{P}_1 - \alpha, \beta - \bar{P}_1\} - \delta \max\{0, P_2 - \alpha, \beta - P_2\}\} dG^\circ(\bar{P}_1). \quad (8)$$

Given this, and the $\{\alpha, \beta\}$ choice of the marketmaker, Agent 1 of type P_1 chooses a search intensity to maximize:

$$\begin{aligned} \max_{M \geq A \geq 0} V_1(A, P_1, B'(\cdot), \alpha, \beta) \\ = \int_{P_2}^{\bar{P}_2} (A^m + (B'(\bar{P}_2))^m) \frac{1}{2} \max\{0, |P_1 - \bar{P}_2| - \delta \max\{0, P_1 - \alpha, \beta - P_1\} - \delta \max\{0, \bar{P}_2 - \alpha, \beta - \bar{P}_2\}\} dF^\circ(\bar{P}_2) \\ - A^n + \delta \max\{0, P_1 - \alpha, \beta - P_1\}. \end{aligned} \quad (9)$$

Similarly, inserting $\{\alpha, \beta\}$ into Agent 2 of type P_2 's problem will give the following objective function:

$$\begin{aligned} \max_{M \geq B \geq 0} V_2(B, P_2, A'(\cdot), \alpha, \beta) \\ = \int_{P_1}^{\bar{P}_1} (B^m + (A'(\bar{P}_1))^m) \frac{1}{2} \max\{0, |\bar{P}_1 - P_2| - \delta \max\{0, \bar{P}_1 - \alpha, \beta - \bar{P}_1\} - \delta \max\{0, P_2 - \alpha, \beta - P_2\}\} dG^\circ(\bar{P}_1) \\ - B^n + \delta \max\{0, P_2 - \alpha, \beta - P_2\}. \end{aligned} \quad (10)$$

Note that the threat points $\delta \max\{0, P_1 - \alpha, \beta - P_1\}$ and $\delta \max\{0, P_2 - \alpha, \beta - P_2\}$ are not multiplied by the matching probability because the agents will get their threat points regardless of whether they trade with each other or with the marketmaker.

If the agents choose to search, they will select the search intensities that solve problems (9) and (10):

$$A^{**}(P_1, \alpha, \beta) = [mR'_1(P_1, \alpha, \beta)/n]^{1/(n-m)} \quad (11)$$

and

$$B^{**}(P_2, \alpha, \beta) = [mR'_2(P_2, \alpha, \beta)/n]^{1/(n-m)}. \quad (12)$$

Note that the marketmaker introduces strategic interaction between the search activities of the agents. Although the search intensity of an agent is still independent of the search intensity of the other agent, it is now dependent on whether or not the other agent searches. This strategic interaction is reflected in the distributions $F^\circ(\cdot)$ and $G^\circ(\cdot)$ in (7) and (8), hence in (11) and (12). Again, the reason for the strategic interaction is the result we derived earlier that an agent cannot contact the other agent if the other agent elects to go to the marketmaker. In the next proposition, we characterize the effect of the marketmaker on the search intensities of the agents.

PROPOSITION 2. *The marketmaker decreases the equilibrium search intensities of both agents.*

Proof. This follows from the fact that $R'_1(P_1, \alpha, \beta)$ of (11) is smaller than $R_1(P_1)$ of (5), and $R'_2(P_2, \alpha, \beta)$ of (12) is smaller than $R_2(P_2)$ of (6). ■

Before we formalize the problem of the marketmaker, note that the two agents will trade with the marketmaker in one of two events: (i) they fail to meet each other (this happens with probability $1 - \theta(\cdot)$), or (ii) they meet but the surplus from trading with the marketmaker is higher for one or both of them. Thus, whether the marketmaker sells and/or buys a unit will depend on the search intensities and the valuations of the agents.

The marketmaker, after solving for Agent 1's and Agent 2's search functions for given values of her ask and bid prices, $A'(\bar{P}_1, \alpha, \beta)$ and $B'(\bar{P}_2, \alpha, \beta)$, respectively, chooses an $\{\alpha, \beta\}$ pair that maximizes her expected profits:

$$\begin{aligned} \max_{\alpha, \beta} \Pi(\alpha, \beta) = & \int_{\alpha}^{\bar{P}_1} \int_{\alpha}^{\bar{P}_2} (1 - \bar{\Theta})(\alpha - I)(\Delta_1^\circ + \Delta_2^\circ) dF(\bar{P}_2) dG(\bar{P}_1) \\ & + \int_{\alpha}^{\bar{P}_1} \int_{P_2}^{\beta} (1 - \bar{\Theta})[(\alpha - I)\Delta_1^\circ + (I - \beta)\Delta_2^\circ] dF(\bar{P}_2) dG(\bar{P}_1) \\ & + \int_{\alpha}^{\bar{P}_1} \int_{\beta}^{\alpha} (1 - \bar{\Theta})(\alpha - I)\Delta_1^\circ dF(\bar{P}_2) dG(\bar{P}_1) \\ & + \int_{P_1}^{\beta} \int_{\alpha}^{\bar{P}_2} (1 - \bar{\Theta})[(\alpha - I)\Delta_2^\circ + (I - \beta)\Delta_1^\circ] dF(\bar{P}_2) dG(\bar{P}_1) \\ & + \int_{P_1}^{\beta} \int_{P_2}^{\beta} (1 - \bar{\Theta})(I - \beta)(\Delta_1^\circ + \Delta_2^\circ) dF(\bar{P}_2) dG(\bar{P}_1) \\ & + \int_{P_1}^{\beta} \int_{\beta}^{\alpha} (1 - \bar{\Theta})(I - \beta)\Delta_1^\circ dF(\bar{P}_2) dG(\bar{P}_1) \end{aligned}$$

$$\begin{aligned}
& + \int_{\beta}^{\alpha} \int_{\alpha}^{\bar{P}_2} (1 - \bar{\Theta})(\alpha - I) \Delta_2^{\circ} dF(\bar{P}_2) dG(\bar{P}_1) \\
& + \int_{\beta}^{\alpha} \int_{\underline{P}_2}^{\beta} (1 - \bar{\Theta})(I - \beta) \Delta_2^{\circ} dF(\bar{P}_2) dG(\bar{P}_1) \\
& + \int_{\alpha}^{\bar{P}_1} \int_{[P_1(1-\delta)+2\delta\alpha]/(1+\delta)}^{P_1} \bar{\Theta} \delta 2(\alpha - I) dF(\bar{P}_2) dG(\bar{P}_1) \\
& + \int_{\alpha}^{\bar{P}_2} \int_{[P_2(1-\delta)+2\delta\alpha]/(1+\delta)}^{P_2} \bar{\Theta} \delta 2(\alpha - I) dG(\bar{P}_1) dF(\bar{P}_2) \\
& + \int_{\underline{P}_1}^{\beta} \int_{P_2}^{[P_2(1-\delta)+2\delta\beta]/(1+\delta)} \bar{\Theta} \delta 2(I - \beta) dG(\bar{P}_1) dF(\bar{P}_2) \\
& + \int_{\underline{P}_1}^{\beta} \int_{P_1}^{[P_1(1-\delta)+2\delta\beta]/(1+\delta)} \bar{\Theta} \delta 2(I - \beta) dF(\bar{P}_2) dG(\bar{P}_1) \tag{13}
\end{aligned}$$

$$\text{s.t. } \alpha \geq 0, \beta \geq 0, A'(\bar{P}_1, \alpha, \beta) \geq 0 \quad \text{and} \quad B'(\bar{P}_2, \alpha, \beta) \geq 0,$$

where I is the marketmaker's valuation of the unit,

$$\Delta_1^{\circ} = \begin{cases} \delta & \text{if } A'(\bar{P}_1, \alpha, \beta) > 0 \\ 1 & \text{if } A'(\bar{P}_1, \alpha, \beta) = 0 \end{cases} \quad \Delta_2^{\circ} = \begin{cases} \delta & \text{if } B'(\bar{P}_2, \alpha, \beta) > 0 \\ 1 & \text{if } B'(\bar{P}_2, \alpha, \beta) = 0, \end{cases}$$

and

$$\bar{\Theta} = \begin{cases} A'(\bar{P}_1, \alpha, \beta)^m + B'(\bar{P}_2, \alpha, \beta)^m & \text{if } A'(\bar{P}_1, \alpha, \beta) > 0 \text{ and } B'(\bar{P}_2, \alpha, \beta) > 0 \\ 0 & \text{otherwise} \end{cases}$$

is the probability by which the marketmaker expects Agent 1 and Agent 2 of (randomly selected) types $\bar{P}_1$ and $\bar{P}_2$ to meet each other.

The explanation for the twelve lines of (13) is given below.

Lines 1–8. If the two parties search and do not meet each other, then their ability to trade with the marketmaker depends on the values of α , β , P_1 , and P_2 . There are nine possibilities in this respect: $P_1 > \alpha$ and $P_2 > \alpha$, $P_1 > \alpha$ and $P_2 < \beta$, $P_1 > \alpha$ and $\alpha > P_2 > \beta$, $P_1 < \beta$ and $P_2 > \alpha$, $P_1 < \beta$ and $P_2 < \beta$, $P_1 < \beta$ and $\alpha > P_2 > \beta$, $\alpha > P_1 > \beta$, and $P_2 > \alpha$, $\alpha > P_1 > \beta$ and $P_2 < \beta$, and $\alpha > P_1 > \beta$ and $\alpha > P_2 > \beta$. If $P_1 > \alpha$ and $P_2 > \alpha$, the marketmaker sells a unit to each agent and gets a surplus of $\delta(\alpha - I)$ per unit ($\alpha - I$ if the agent goes directly to the marketmaker). This is reflected in Line 1. If $P_1 > \alpha$ and $P_2 < \beta$, the marketmaker sells a unit to Agent 1 and realizes a surplus of $\delta(\alpha - I)$ ($\alpha - I$ if Agent 1 goes to her directly) and buys a unit from Agent 2 and receives a surplus of $\delta(I - \beta)$ ($I - \beta$ if Agent 2 goes to her directly). This is reflected in Line 2. The next six lines have a

similar interpretation. Note that if $\alpha > P_i > \beta$, then the marketmaker will not be able to trade with agent i . This also explains why the last case, $\alpha > P_1 > \beta$ and $\alpha > P_2 > \beta$, has not been included in (13), because the result would be zero surplus for the marketmaker.

Lines 9–12. If the two agents meet, then they will enjoy a higher surplus by trading with each other in all of the first four cases specified in the Appendix (note that, in all of these four cases, the surplus each agent gets by trading with the other agent exceeds the surplus he would get by trading with the marketmaker). It is only in cases 5 and 6 that the agents might prefer trading with the marketmaker. In case 5, in which $P_1 > P_2 > \alpha > \beta$ or $P_2 > P_1 > \alpha > \beta$, the agents will choose to trade with the marketmaker only if $P_1 - P_2 \leq \delta(P_1 - \alpha) + \delta(P_2 - \alpha)$ or $P_2 - P_1 \leq \delta(P_1 - \alpha) + \delta(P_2 - \alpha)$, respectively. The former implies $P_2 \geq [P_1(1 - \delta) + 2\delta\alpha]/(1 + \delta)$, while the latter implies $P_1 \geq [P_2(1 - \delta) + 2\delta\alpha]/(1 + \delta)$. These are captured by the lower limits of the inside integrals of Lines 9 and 10. Similarly, in case 6 of the Appendix, where $\alpha > \beta > P_1 > P_2$ or $\alpha > \beta > P_2 > P_1$, the agents will choose to trade with the marketmaker only if $P_1 - P_2 \leq \delta(\beta - P_1) + \delta(\beta - P_2)$ or $P_2 - P_1 \leq \delta(\beta - P_1) + \delta(\beta - P_2)$, respectively. The former implies $P_1 < [P_2(1 - \delta) + 2\delta\beta]/(1 + \delta)$, while the latter implies $P_2 < [P_1(1 - \delta) + 2\delta\beta]/(1 + \delta)$. These are captured by the lower limits of the inside integrals of Lines 11 and 12.

The Lagrangian function for (13) is given by

$$\mathcal{L}(\alpha, \beta) = \Pi(\alpha, \beta) + \lambda A'(\cdot) + \mu B'(\cdot).$$

The Kuhn–Tucker conditions are

$$\begin{aligned} \Pi_\alpha + \lambda A'_\alpha + \mu B'_\alpha &\leq 0, \alpha \geq 0 & \text{and} & \quad \alpha \mathcal{L}_\alpha = 0 \\ \Pi_\beta + \lambda A'_\beta + \mu B'_\beta &\leq 0, \beta \geq 0 & \text{and} & \quad \beta \mathcal{L}_\beta = 0 \\ A' &\geq 0, \lambda \leq 0 & \text{and} & \quad A'\lambda = 0 \\ B' &\geq 0, \mu \leq 0 & \text{and} & \quad B'\mu = 0, \end{aligned}$$

where the subscripts denote the corresponding partial derivatives and λ and μ are the Lagrangian multipliers.⁷

⁷ Note that λ and μ also measure how the optimal value of $\Pi(\alpha, \beta)$ reacts to a slight relaxation in the constraints $B''(\bar{P}_2, \alpha, \beta) \geq 0$ and $A''(\bar{P}_1, \alpha, \beta) \geq 0$. If, for example $A''(\bar{P}_1, \alpha, \beta) \geq 0$ is *optimally* not binding, then relaxing it will not affect the optimal value of $\Pi(\alpha, \beta)$. On the other hand, if a slight relaxation in the constraint does increase $\Pi(\alpha, \beta)$, then that constraint must in fact be binding in the optimal solution.

Clearly, we will have $\alpha^* > \beta^*$, where α^* and β^* are the optimum ask and bid prices. Otherwise there would be arbitrage opportunities for the agents and negative expected profits for the marketmaker. If the profit-maximization yields $\alpha^* < \beta^*$, the marketmaker can simply set $\alpha^* > \max\{\bar{P}_1, \bar{P}_2\}$ and $\beta^* < \min\{\underline{P}_1, \underline{P}_2\}$ (and obtain zero profits), which is the equivalent of saying that no marketmaking services will be provided in that market. We now define equilibrium.

The *Bayesian-Nash Equilibrium* under the marketmaker is a triple of strategies $\{(\alpha^*, \beta^*), A^{**}(\cdot, \alpha^*, \beta^*), B^{**}(\cdot, \alpha^*, \beta^*)\}$ such that:

- (i) $A^{**}(P_1, \alpha^*, \beta^*)$ maximizes Agent 1's expected return from search given his conjecture is $B^{**}(\cdot, \alpha^*, \beta^*)$ and given the (α^*, β^*) choice of the marketmaker, and thus satisfies (11) $\forall P_1$,
- (ii) $B^{**}(P_2, \alpha^*, \beta^*)$ maximizes Agent 2's expected return from search given his conjecture is $A^{**}(\cdot, \alpha^*, \beta^*)$ and given the (α^*, β^*) choice of the marketmaker, and thus satisfies (12) $\forall P_2$,
- (iii) $\{\alpha^*, \beta^*\}$ maximizes the marketmaker's expected profits given her conjecture of the optimal reactions of Agent 1 and Agent 2 are $A^{**}(\cdot, \alpha^*, \beta^*)$ and $B^{**}(\cdot, \alpha^*, \beta^*)$, and thus solves the problem in (13).

The existence of the Bayesian-Nash Equilibrium for this game has been proved in Yavaş (1990a). As indicated in that study, there might be multiple equilibria, involving more than one equilibrium search intensity function for each agent for some values of $\{\alpha, \beta\}$, depending on the distributions $F(\cdot)$ and $G(\cdot)$ and the parameters δ , m , and n . We confine our attention here to the economies that yield unique equilibrium.⁸ The role of middlemen in economies exhibiting multiple equilibria has been studied in detail in Yavaş (1990b).

We now turn to the case in which the middleman is a matchmaker.

⁸ As an example of an intermediated search market with a unique equilibrium, consider the case in which $\delta = 1$ (agents are very patient). Now, it is a weakly dominant strategy for each agent to search or to wait to be contacted before going to the marketmaker, because he is not penalized for going to the marketmaker after having searched or waited to be contacted. Given that the marginal cost of zero search is zero, while the marginal benefit of zero search is infinite, and given that an agent can contact the other agent even if the other agent chooses to wait to be contacted, the equilibrium will consist of both agents searching for each other before going to the marketmaker. When $\delta < 1$, however, there might be more than one equilibrium, i.e., more than one pair of F° and G° functions which determine the set of agent types who search and who go directly to the marketmaker, depending on the conjectures of the agents and the parameters of the model. Although one can provide examples of economies where the equilibrium is unique, the complexity of the problem does not allow us to specify the whole set of parameters for which there is a unique equilibrium. The reason for confining attention here to the economies with a unique equilibrium is to avoid additional problems created by the multiplicity of equilibria (e.g., equilibrium selection, coordination failures).

c. *Matchmaker*

Unlike the marketmaker, a matchmaker does not sell and buy on her own account. She only matches the two agents. That is, the matchmaker improves the probability that agents secure a match but, unlike the marketmaker, does not create new trades.

An agent first decides whether to buy the matching service of the matchmaker or to search for the other agent. If both agents go to the matchmaker, they get matched through her and they share equally the surplus. In return, the matchmaker charges each agent $0 \leq k \leq 1$ portion of the transaction price.⁹ However, the agents do not have to pay commission fees to the matchmaker if they choose not to trade with each other.

The agents can also go to the matchmaker after they have already searched (but failed to meet each other). As in the case of the marketmaker, if the agents go to the matchmaker after having searched, their surplus is discounted by δ , $0 \leq \delta \leq 1$.

It will be assumed that once one of the agents goes to the matchmaker, he cannot be contacted by the other agent. In order for a match to occur the other agent also must go to the matchmaker. In other words, the match must go through the matchmaker, and each side must pay a fraction k of the transaction price to the matchmaker.¹⁰ This institutional design is referred to as "exclusive dealership" in the rest of the paper. The alternative design is one in which the two agents can contact each other on their own even after one of them has already gone to the matchmaker. This alternative model will be studied and compared with the "exclusive dealership" design in subsequent work.

As in the case of the marketmaker, the matchmaker has an endowment of the commodity, and she values the unit at I .¹¹ She knows the distributions $F(\cdot)$ and $G(\cdot)$ but not the actual valuations of the agents. The matchmaker acts first and chooses k at the beginning of the period. Agents know k before they choose their search intensities.

It has been shown in Proposition 1 that the marketmaker reduces the price dispersion by providing an alternative source of trade for the agents

⁹ There are alternative ways of charging the agents for buying the matchmaker's services, such as a fixed fee per transaction (marriage markets) or charging only one side a percentage of the transaction price (housing markets). These issues will be dealt with in a separate paper, in which different institutional designs of matchmaking will be compared.

¹⁰ In the housing markets, once the seller lists his house with a real estate broker, he will have to pay a certain percentage of the transaction price to the broker even if he is contacted directly by the buyer or contacts the buyer on his own. This assumption also makes match-making more comparable to the marketmaking presented in Section IIb.

¹¹ The matching service can also be provided without this endowment. The purpose of endowing the matchmaker is to keep the matchmaking institution comparable to the market-making institution when we compare the two in terms of the profits to intermediation and in terms of the welfare effects.

at the prices α and β . The following proposition shows that even though the matchmaker simply matches the two agents, rather than offering an alternative source of trade, she also affects the price dispersion.

PROPOSITION 3. *The matchmaker reduces the price dispersion of the matches that she facilitates.*

Proof. Suppose that the valuations of the agents in a match are $P_i > P_j$, $i = 1, 2$, $j = 1, 2$, and $i \neq j$. If the agents form the match by searching for each other, then each agent's reservation price is simply his valuation, P_i or P_j . If the agents are matched through the matchmaker, however, they must pay commission fees to the matchmaker (recall that the agents must pay commission fees only if they choose to trade with each other). Hence, agent i (j) will choose to buy (sell) at price P_i^* (P_j^*) iff his surplus from trade exceeds the commission fee: $P_i - P_i^* \geq kP_i^*$ ($P_j - P_j^* \geq kP_j^*$). The reservation prices of the two agents become $P_i^* = P_i/(1 + k)$ and $P_j^* = P_j/(1 - k)$. Clearly, the matchmaker increases the seller's reservation price, since $P_j^* \geq P_j$, and decreases the buyer's reservation price, since $P_i^* \leq P_i$. Therefore, the range of prices at which a trade can take place through the matchmaker is narrower than the range that would prevail if there were no matchmaker. ■

As is true of the marketmaker, it is also possible that the matchmaker might induce some Agent 1 and Agent 2 types not to search, but rather to go directly to her. Because of the exclusive dealership assumption, Agent 1 (if he searches) will meet only those Agent 2 types who search. Similarly, Agent 2 (if he searches) will meet only those Agent 1 types who search. Hence, we cannot use the distributions $F(\cdot)$ and $G(\cdot)$ to calculate the expected gains of the agents from search. Instead, we will use distributions $\hat{F}(\cdot)$ and $\hat{G}(\cdot)$, where

$$\begin{aligned} \hat{F}(\tilde{P}_2) &= \begin{cases} F(\tilde{P}_2) & \text{if } B''(\tilde{P}_2) > 0 \\ 0 & \text{if } B''(\tilde{P}_2) = 0 \end{cases} & \text{and} \\ \hat{G}(\tilde{P}_1) &= \begin{cases} G(\tilde{P}_1) & \text{if } A''(\tilde{P}_1) > 0 \\ 0 & \text{if } A''(\tilde{P}_1) = 0, \end{cases} \end{aligned}$$

with $B''(\tilde{P}_2)$ as Agent 1's conjecture about the search intensity of Agent 2 of type $\tilde{P}_2$ and $A''(\tilde{P}_1)$ as Agent 2's conjecture about the search intensity of Agent 1 of type $\tilde{P}_1$.

Given k and given his conjecture about the search intensity function of Agent 2, if Agent 1 of type P_1 chooses to search, he will choose a search intensity to maximize

$$\max_{M \geq A \geq 0} W_1(A, B''(\cdot), k)$$

$$= \int_{P_2}^{\bar{P}_2} (A^m + (B''(\bar{P}_2))^m)^{\frac{1}{2}} |P_1 - \bar{P}_2| d\hat{F}(\bar{P}_2) - A^n \\ + \int_{P_2}^{\bar{P}_2} [1 - A^m - (B''(\bar{P}_2))^m] \delta^{\frac{1}{2}} \max\{0, |P_1 - \bar{P}_2| - k(P_1 + \bar{P}_2)\} dF(\bar{P}_2), \quad (14)$$

where $\frac{1}{2}(P_1 + P_2)$ is the transaction price,¹² and $\frac{1}{2}k(P_1 + P_2)$ is the commission fee paid to the matchmaker. The explanation for the last term in (14) is that with probability $1 - A^m - B^m$, the search efforts of the agents will fail to result in a meeting and they will both go to the matchmaker. They will do so because once search is done, the search cost is sunk and the expected return from going to the matchmaker is positive for both agents. Hence, it pays for both agents to go to the matchmaker if they do not meet through the search process.¹³ This also implies that Agent 1 might meet any Agent 2 type when he goes to the matchmaker. That is why the distribution $F(\cdot)$, and not $\hat{F}(\cdot)$, has been used in the second integral of (14).

The solution to (14), $A^{***}(P_1, k)$, is given by

$$A^{***}(P_1, k) = [m[\hat{R}_1(P_1) - \delta R_1(P_1, k)]/n]^{1/(n-m)}, \quad (15)$$

where

$$\hat{R}_1(P_1) = \int_{P_2}^{\bar{P}_2} \frac{1}{2} |P_1 - \bar{P}_2| d\hat{F}(\bar{P}_2) \quad \text{and} \\ R_1(P_1, k) = \int_{P_2}^{\bar{P}_2} \frac{1}{2} \max\{0, |P_1 - \bar{P}_2| - k(P_1 + \bar{P}_2)\} dF(\bar{P}_2). \quad (16)$$

Similarly, Agent 2 of type P_2 solves

¹² To see this, let T be the transaction price and let (without loss of generality) $P_1 > P_2$. Agents' total net surplus from a trade through the matchmaker is $(P_1 - P_2) - kT - kT$. The bargaining solution gives each agent half of this surplus; $\frac{1}{2}(P_1 - P_2) - kT$. We also know that a transaction price of T generates a surplus of $(P_1 - T) - kT$ for Agent 1 and of $(T - P_2) - kT$ for Agent 2. Equating these two surpluses to that derived from the bargaining solution requires $T = \frac{1}{2}(P_1 + P_2)$.

¹³ Remember that an agent does not pay the matchmaker anything if he does not trade with the other agent. This assures the agents that they have nothing to lose by going to the matchmaker after they have failed to meet each other. Hence, it is a weakly dominant strategy for both agents to go to the matchmaker if their search efforts do not yield a match.

$$\begin{aligned}
& \max_{M \geq B \geq 0} W_2(B, A''(\cdot), k) \\
&= \int_{P_1}^{\bar{P}_1} (B^m + (A''(\bar{P}_1))^m) \frac{1}{2} |\bar{P}_1 - P_2| d\hat{G}(\bar{P}_1) - B^n \\
&+ \int_{P_1}^{\bar{P}_1} [1 - B^m - (A''(\bar{P}_1))^m] \delta \frac{1}{2} \max\{0, |\bar{P}_1 - P_2| - k(\bar{P}_1 + P_2)\} dG(\bar{P}_1).
\end{aligned} \tag{17}$$

The solution satisfies

$$B^{***}(P_2, k) = [m[\hat{R}_2(P_2) - \delta R_2(P_2, k)]/n]^{1/(n-m)}, \tag{18}$$

where

$$\begin{aligned}
\hat{R}_2(P_2) &= \int_{P_1}^{\bar{P}_1} \frac{1}{2} |\bar{P}_1 - P_2| d\hat{G}(\bar{P}_1) \quad \text{and} \\
R_2(P_2, k) &= \int_{P_1}^{\bar{P}_1} \frac{1}{2} \max\{0, |\bar{P}_1 - P_2| - k(\bar{P}_1 + P_2)\} dG(\bar{P}_1).
\end{aligned} \tag{19}$$

Note from (15) and (18) that the matchmaker (as well as the marketmaker) introduces strategic interaction between agents' actions. Although the search intensity of Agent 1 is independent of the search intensity of Agent 2, it is dependent on whether or not Agent 2 searches. Therefore, Agent 1 must conjecture about which Agent 2 types will search and which Agent 2 types will go directly to the matchmaker. Similarly, Agent 2 forms his conjecture about which Agent 1 types will search and which Agent 1 types will choose to go to the matchmaker. This is captured by $\hat{F}(\cdot)$ in (15) and by $\hat{G}(\cdot)$ in (18). The source of the strategic interaction here is the assumption of exclusive dealership—an agent cannot contact the other agent by searching if the other agent has already gone to the matchmaker. Without exclusive dealership, an agent could contact all types of the other agent, thereby precluding any strategic interaction. In the next proposition, we characterize the effect of the matchmaker on the search intensities of the agents.

PROPOSITION 4. *The matchmaker decreases the equilibrium search intensities of both agents.*

Proof. This follows from the fact that $\hat{R}_1(P_1) - \delta R_1(P_1, k)$ of (15) is less than $R_1(P_1)$ of (5), and $\hat{R}_2(P_2) - \delta R_2(P_2, k)$ of (18) is less than $R_2(P_2)$ of (6). ■

The matchmaker, after solving for the agents' reaction functions $A''(\cdot)$ and $B''(\cdot)$ for given values of k , chooses the optimal k that maximizes her profits:

$$\max_{1 \geq k \geq 0} \pi(k) = \int_{\bar{P}_1}^{\bar{P}_1} \int_{\bar{P}_2}^{\bar{P}_2} [1 - (A''(\cdot))^m - (B''(\cdot))^m] k (\bar{P}_1 + \bar{P}_2) \Delta_m(\bar{P}_2, \bar{P}_1) Q(\bar{P}_1, \bar{P}_2, k) dF(\bar{P}_2) dG(\bar{P}_1), \quad (20)$$

where $A''(\cdot) = [m[\hat{R}_1(\bar{P}_1) - \delta R_1(\bar{P}_1, k)]/n]^{1/(n-m)}$, $B''(\cdot) = [m[\hat{R}_2(\bar{P}_2) - \delta R_2(\bar{P}_2, k)]/n]^{1/(n-m)}$,

$$\Delta_m(\bar{P}_2, \bar{P}_1) = \begin{cases} 1 & \text{if } A''(\bar{P}_1, k) = B''(\bar{P}_2, k) = 0 \\ \delta & \text{otherwise,} \end{cases}$$

and

$$Q(\bar{P}_1, \bar{P}_2, k) = \begin{cases} 1 & \text{if } |\bar{P}_1 - \bar{P}_2| \geq k(\bar{P}_1 + \bar{P}_2) \\ 0 & \text{otherwise.} \end{cases}$$

Note that the matchmaker's valuation of the unit, I , has no impact on her profits. This is in contrast to the case of the marketmaker whose valuation is crucial in determining her pricing policy and profits. This is because the matchmaker simply matches the two agents, without ever trading her unit. We now define equilibrium in the matchmaker case.

The Bayesian–Nash Equilibrium under the matchmaker is a triple of strategies $\{k, A^{***}(\cdot, k^*), B^{***}(\cdot, k^*)\}$ which satisfy the following conditions:

(i) $A^{***}(P_1, k^*)$ maximizes Agent 1's expected return from search given his conjecture is $B^{***}(\cdot, k^*)$ and given the k^* choice of the matchmaker, and thus satisfies (15) $\forall P_1$,

(ii) $B^{***}(P_2, k^*)$ maximizes Agent 2's expected return from search given his conjecture is $A^{***}(\cdot, k^*)$ and given the k^* choice of the matchmaker, and thus satisfies (18) $\forall P_2$,

(iii) k^* maximizes the matchmaker's profits given her conjecture of the optimal reactions of Agent 1 and Agent 2, $A^{***}(\cdot, k^*)$ and $B^{***}(\cdot, k^*)$, and thus solves (20).

The existence of the Bayesian–Nash Equilibrium in the presence of a matchmaker has been proved in Yavaş (1991a). As in Section IIb, there might be multiple equilibria, depending on the distributions $F(\cdot)$ and $G(\cdot)$ and the parameters δ , m , and n . Again, we confine our attention here to

the economies that yield a unique equilibrium. Discussion in Footnote 8 on the multiplicity of equilibria also applies to the case in which the middleman is a matchmaker.

We now turn to a comparison of the two institutions of intermediation.

d. *Comparison*

To analyze the middleman's choice of intermediation mode, we need to compare the expected profits under the two types of middlemen. We will also explore the welfare implications of this choice. This will explain whether a central planner would choose to have a marketmaker or a matchmaker in a market.

It has been shown in Yavaş (1991b) that when the valuations of the agents are common knowledge, the middleman would choose to be a marketmaker rather than a matchmaker, and that welfare is higher with the marketmaker. How can this be reconciled with the existence of matchmakers in many markets? One explanation is that in some markets, such as labor markets and marriage markets, the nature of the commodity makes it impossible for the middleman to buy or sell on her own account, thus ruling out marketmaking. In this section, we provide a second explanation. We show that *even if the nature of the commodity allows market-making as well as matchmaking*, the middleman might choose matchmaking over marketmaking when she does not know the valuations of the agents. Furthermore, the welfare impact of the matchmaker might exceed that of the marketmaker when the valuations are private information.

First, we compare the two types of middlemen in terms of their expected profits. It is shown that whether the marketmaker receives more or less expected profit than the matchmaker depends on the parameters of the model, I , δ , n , and m , and the distribution functions $F(\cdot)$ and $G(\cdot)$.

The valuation of the middleman, I , relative to the distribution of valuations of the agents plays an important role in the comparison of the expected profits of the two middleman types. Consider the following: $\bar{P}_2 \in [4, 6]$, $\bar{P}_1 \in [4, 5]$, and $I = 1$. This gives an expected surplus of less than 2 for any match, so the expected profits of the matchmaker cannot exceed 2. Now, one option that the marketmaker has (which may not even be the profit maximizing choice) is simply to set $\alpha = \bar{P}_1 = 4$ (and $\beta \leq 4$), in which case the agents will not search but will go directly to the marketmaker. The marketmaker then sells a unit to each agent and enjoys a surplus of $2(\alpha - I) = 6$ (at the equilibrium values of α and β , the profit of the marketmaker might be even greater than 6). Similarly, if I is changed from 1 to, let us say, 9, the marketmaker can still earn at least 6 units of profit by setting $\beta = 6$ (and $\alpha \geq 6$), whereas the matchmaker's profit will still be less than 2. A generalization of this result appears in the proposition below.

PROPOSITION 5. *The middleman will choose to be a marketmaker if (i) $I \leq \min\{\underline{P}_1, \underline{P}_2\}$ and $2(\min\{\underline{P}_1, \underline{P}_2\} - I) \geq \max\{\bar{P}_1, \bar{P}_2\} - \min\{\underline{P}_1, \underline{P}_2\}$, or (ii) $I \geq \max\{\bar{P}_1, \bar{P}_2\}$ and $2(I - \max\{\bar{P}_1, \bar{P}_2\}) \geq \max\{\bar{P}_1, \bar{P}_2\} - \min\{\underline{P}_1, \underline{P}_2\}$.*

Proof. As is shown in the example above, the marketmaker can make at least $2(\min\{\underline{P}_1, \underline{P}_2\} - I)$ units of profit by setting $\alpha = \min\{\underline{P}_1, \underline{P}_2\}$ in case (i) and at least $2(I - \max\{\bar{P}_1, \bar{P}_2\})$ units of profit by setting $\beta = \max\{\bar{P}_1, \bar{P}_2\}$ in case (ii). In both cases, the profit of the marketmaker will exceed the highest possible level of profit for the matchmaker, $\max\{\bar{P}_1, \bar{P}_2\} - \min\{\underline{P}_1, \underline{P}_2\}$. ■

This result is a consequence of the ability of the marketmaker to sell without buying or to buy without selling, as well as to buy and sell. The matchmaker, on the contrary, can only match Agent 1 with Agent 2, i.e., she can only buy *and* sell a unit. Condition (i) in Proposition 5 indicates that if the middleman's valuation is too low relative to the valuations of the agents, then it is more profitable to sell a unit without buying than to buy and sell one. Hence, a marketmaker can earn more profit than a matchmaker simply by selling her endowment. Similarly, condition (ii) indicates that if the middleman's valuation is too high relative to the valuations of the agents, then it is more profitable to buy a unit without selling than to buy and sell one. Hence, a marketmaker can earn more profit than a matchmaker simply by buying a unit. Again, this result is due to the ability of the marketmaker to take only one side of the market.

Following the same reasoning, it is trivial to show that the marketmaker's profits exceed the matchmaker's profit if the expected surplus from a match for both agents is (almost) equal to zero, i.e., $\underline{P}_1 \approx \underline{P}_2 \approx \bar{P}_1 \approx \bar{P}_2$.

PROPOSITION 6. *Let $\delta > 0$, $\bar{P}_2 > \underline{P}_1$, and $\bar{P}_1 > \underline{P}_2$. The middleman will choose to be a marketmaker for m close to zero.*

Proof. It can be shown that as m goes to zero, $\pi(k^*)$ goes to zero, while $\Pi(\alpha^*, \beta^*)$ goes to

$$\begin{aligned}
 & + \int_{\alpha}^{\bar{P}_1} \int_{[P_1(1-\delta)+2\delta\alpha]/(1+\delta)}^{P_1} \delta 2(\alpha - I) dF(\tilde{P}_2) dG(\tilde{P}_1) \\
 & + \int_{\alpha}^{\bar{P}_2} \int_{[P_2(1-\delta)+2\delta\alpha]/(1+\delta)}^{P_2} \delta 2(\alpha - I) dG(\tilde{P}_1) dF(\tilde{P}_2) \\
 & + \int_{\underline{P}_2}^{\beta} \int_{P_2}^{[P_2(1-\delta)+2\delta\beta]/(1+\delta)} \delta 2(I - \beta) dG(\tilde{P}_1) dF(\tilde{P}_2) \\
 & + \int_{\underline{P}_1}^{\beta} \int_{P_1}^{[P_1(1-\delta)+2\delta\beta]/(1+\delta)} \delta 2(I - \beta) dF(\tilde{P}_2) dG(\tilde{P}_1) > 0. \quad \blacksquare
 \end{aligned}$$

The intuition is that as m goes to zero, search becomes very efficient and the probability of a match, $A^m + B^m$, goes to one. This, in turn, drives the expected profits of the matchmaker to zero. The marketmaker, on the other hand, can still enjoy positive profits by offering a better deal to one or both of the agents than what they would obtain if they traded with each other. This is exactly what the last four lines of the marketmaker's profit function (13) represent, and the profit of the marketmaker given above is equal to those four lines as m goes to zero.

The following proposition shows, however, that the middleman will choose to be a matchmaker if search is very costly and inefficient.

PROPOSITION 7. *Let $I = \frac{1}{2}$, $\delta = 1$, and $G(\cdot)$ and $F(\cdot)$ be uniform with $\tilde{P}_1 \in [1, 2]$ and $\tilde{P}_2 \in [0, .01]$. Then, the matchmaker's expected profit will exceed the marketmaker's for m close to n .*

Proof. As m goes to n , the probability of a match between Agent 2 and Agent 1 goes to zero. If the matchmaker charges a commission rate of $k = 0.98$, then all the matches between $\tilde{P}_1$ and $\tilde{P}_2$ will be traded through the matchmaker (to see this, note that $P_1 - 0.01 \geq 0.98(P_1 + 0.01) \forall P_1$ and $1 - P_2 \geq 0.98(P_2 + 1) \forall P_2$). Since $k = 0.98$ may not necessarily be the profit-maximizing k , the expected profit of the matchmaker will be:

$$\pi(k^*) \geq \int_1^2 \int_0^{0.01} 0.98(\tilde{P}_1 + \tilde{P}_2) dF(\tilde{P}_2) dG(\tilde{P}_1) = 1.4749 = \pi(0.98).$$

As the matching probability $\theta(\cdot)$ goes to zero, the profit of the marketmaker, on the other hand, becomes $\Pi(\alpha, \beta) = (1 - F(\alpha))(\alpha - I) + G(\beta)(I - \beta)$ (see 13)). The solution is given by $\alpha^* = 1 + I/2 = 1.25$ and $\beta^* = 0.01$. Substituting these back into the profit function of the marketmaker yields the maximum profit of $\Pi(\alpha^*, \beta^*) = 1.0525 < \pi(k^*)$. Hence, it is more profitable to be a matchmaker than to be a marketmaker. ■

The intuition here is that the matchmaker will receive (almost) all of the surplus from a match regardless of the realization of the valuations of the agents, whereas the marketmaker will not have a trade with Agent 1 if that agent's valuation turns out to be less than α^* and she will not have a trade with Agent 2 if that agent's valuation turns out to be bigger than β^* . Further, she will not receive all of the surplus unless the realization of Agent 1's and Agent 2's valuations are exactly α^* and β^* , respectively.

These propositions illustrate that the middleman will choose to provide the marketmaking service if her valuation is too high or too low compared to the valuations of the agents, or if search is effectively costless; and that she will prefer providing the match-making service if search is very costly and inefficient.

We now return to the second question: which type of middleman will emerge in a market if a central planner makes the choice? The answer to this question depends on which type of middleman is expected to have a bigger welfare effect. We use total expected surplus (Agent 1's expected surplus + Agent 2's expected surplus + the middleman's expected profits) to measure welfare.

It turns out that the determination of which surplus is higher depends on the exogenous parameters of the model. We start with the case where the matchmaker generates a bigger welfare effect. For that, we consider the economy described in Proposition 7 where $I = \frac{1}{2}$, $\delta = 1$, and $G(\cdot)$ and $F(\cdot)$ uniform with $\tilde{P}_1 \in [1, 2]$ and $\tilde{P}_2 \in [0, 0.01]$. As in Proposition 7, as m approaches n , the search intensities of the agents go to zero. Hence, the probability of a match between Agent 1 and Agent 2 goes to zero and so do the search costs. Given that $\delta = 1$, the agents will go to the matchmaker and obtain a match if they do not meet each other. Further, their surplus will not be discounted. It has been shown in Proposition 7 that the matchmaker's expected profit will be $\pi(k^*) \geq 1.4749$. Hence, the total expected surplus with the matchmaker is $TS(k) \geq 1.4749$.

As the matching probability $\theta(\cdot)$ goes to zero, the total expected surplus in the case of a the marketmaker, on the other hand, becomes $TS(\alpha, \beta) = 1.0525$ (marketmaker's expected profit) + 0.25 (Agent 1's expected surplus) + 0.005 (Agent 2's expected surplus) = 1.3075 . Consequently, $TS(k) > TS(\alpha, \beta)$.

The intuitive reason for this result is that when the agents do not meet, they will secure a certain match by going to the matchmaker. With the marketmaker, on the other hand, a trade is not certain. Agent 2 will be able to trade with the marketmaker with probability $1 - F(\alpha)$, i.e., if $P_2 > \alpha$, and Agent 1 will be able to sell his unit to the marketmaker with probability $G(\beta)$, i.e., if $\beta > P_1$. Note that the middleman's choice coincides with that of the planner when search is very inefficient (when m and n are very close to 1).

Next, we provide an example where the marketmaker produces a higher welfare effect than the matchmaker. To do so, we return to Proposition 5 above where $\tilde{P}_2 \in [4, 6]$, $\tilde{P}_1 \in [4, 5]$, and $I = 1$. Since the difference $\tilde{R}(\tilde{P}_1, \tilde{P}_2) - \text{search costs}$ is the total expected surplus that Agent 1, Agent 2, and the matchmaker share, the total expected surplus under the matchmaker will be less than 2 units. As shown in Proposition 5, the marketmaker, on the other hand, can produce a surplus of at least 6 units simply by offering her unit for sale at $\alpha = \underline{P}_1 = 4$. Hence, the welfare effect of the marketmaker will be greater than that of the matchmaker. Following Proposition 5, this result can be generalized to those markets where (i) $I \leq \min\{\underline{P}_1, \underline{P}_2\}$ and $2(\min\{\underline{P}_1, \underline{P}_2\} - I) \geq \max\{\bar{P}_1, \bar{P}_2\} - \min\{\underline{P}_1, \underline{P}_2\}$, or (ii) $I \geq \max\{\bar{P}_1, \bar{P}_2\}$ and $2(I - \max\{\bar{P}_1, \bar{P}_2\}) \geq \max\{\bar{P}_1, \bar{P}_2\} -$

$\min\{\underline{P}_1, \underline{P}_2\}$. Again, this is a result of the ability of the marketmaker to sell and/or buy. The matchmaker, on the contrary, can only buy *and* sell. Following the same reasoning, it is trivial to show that the marketmaker dominates the matchmaker if the expected surplus from a match is equal to zero.

Note again that the planner's and the middleman's choice are the same in these two cases. They both choose marketmaking over matchmaking when the valuation of the middleman is much bigger or smaller than that of the agents or when the surplus that the agents obtain by trading with each other is zero.

III. CONCLUDING REMARKS

It has been shown that marketmaking might yield higher or lower profits and welfare effects than matchmaking, depending on the efficiency and the cost of search and on the distribution of valuations of the agents. If, for example, search is efficient and costless, then the middleman prefers to be a marketmaker. If, on the other hand, search is relatively inefficient and sufficiently costly, then matchmaking leads to higher profits and welfare. This finding is consonant with the stylized fact that we observe matchmakers in housing markets where the search for a buyer or seller of a house with the desired attributes can be costly and inefficient, while we observe marketmakers in the stock market where the search for a trader of a particular stock is relatively more efficient and less costly.

The model presented in this paper focuses on the role that the characteristics of search markets play in explaining whether the intermediary is a marketmaker or a matchmaker. However, in some markets other factors (e.g., inventory costs, insider trading) may also impinge on this choice. Hence, the model here cannot be used to explain the existence of marketmakers versus matchmakers in *all* exchange markets. It also could be the case that there is a fixed cost of intermediation and that this fixed cost differs between marketmaking and matchmaking services in a particular market. For some commodities it might be less costly to provide the service of marketmaking, while for others it might be cheaper to supply a matching service. In that case, these cost differentials would also affect the type of middleman extant.

Although the middleman in this analysis is a monopolist, one can interpret the search market described in the model as the competition that the middleman faces. The cost and efficiency of search can proxy for the intensity of this competition. For the specialist in the stock market, for example, the search market can be interpreted as the competition from the upstairs markets.

A middleman may provide both marketmaking and matchmaking services. An example of this is the specialist in the stock market. As an intermediary, the specialist not only sells and buys at the ask and bid prices that she sets, but also takes sell orders and buy orders with which she matches traders. Assigning both the marketmaking and matchmaking roles to a middleman adds a new dimension to the problem. The middleman can now use these sell and buy orders (as a matchmaker) to extract information about the demand and supply in the market and can use this information in choosing the ask and bid prices (as a marketmaker). The pricing problem of the specialist has been studied by Garman (1976), and an explicit consideration has been given to the informational dimension of the problem in Gould and Verrecchia (1985).

Here, attention has been confined to the search economies that display unique equilibria. Some search economies, however, might display multiple equilibria. The role of the two types of middlemen in such economies is examined in Yavaş (1990b). There it is shown that the two types of middleman can also have an equilibrium selection role. It is also shown that the marketmaker has a coordination role: she eliminates some/all of the Pareto-dominated search equilibria.

The middleman has been introduced exogenously to the economy in Sections IIb and IIc of this paper. An interesting extension would be to start with three agents in Section IIa and then to analyze the conditions under which one of the agents will arise as a middleman endogenously. This question is important because the observed welfare effect depends on which agent emerges as the middleman. Answering this question might also enable us to determine the optimal mechanism for implementing the "right" agent as the middleman.

APPENDIX

This appendix discusses how the two agents share the surplus when they meet using the bargaining rule assumed in Section IIa of this paper. Since the no-arbitrage condition yields $\alpha \geq \beta$, there will be six possible cases:

1. $P_i > \alpha > \beta > P_j$: Agent j gets $\delta(\beta - P_j) + \frac{1}{2}[P_i - P_j - \delta(\beta - P_j) - \delta(P_i - \alpha)]$ and Agent i 's share is $\delta(P_i - \alpha) + \frac{1}{2}[P_i - P_j - \delta(\beta - P_j) - \delta(P_i - \alpha)]$.
2. $P_i > \alpha > P_j > \beta$: Agent j receives $\frac{1}{2}[P_i - P_j - \delta(P_i - \alpha)]$ and Agent i gets $\delta(P_i - \alpha) + \frac{1}{2}[P_i - P_j - \delta(P_i - \alpha)]$.
3. $\alpha > P_i > \beta > P_j$: Agent j 's share is $\delta(\beta - P_j) + \frac{1}{2}[P_i - P_j - \delta(\beta - P_j)]$ and Agent i gets $\frac{1}{2}[P_i - P_j - \delta(\beta - P_j)]$.
4. $\alpha > P_i > P_j > \beta$: Each agent gets $\frac{1}{2}(P_i - P_j)$.

Note that in all of these four cases both parties are better off when they trade with each other.

5. $P_i > P_j > \alpha > \beta$: The two agents will choose to trade with each other only if $P_i - P_j \geq \delta(P_i - \alpha) + \delta(P_j - \alpha)$ in which case Agent j receives $\delta(P_j - \alpha) + \frac{1}{2}[P_i - P_j - \delta(P_i - \alpha) - \delta(P_j - \alpha)]$ and Agent i gets $\delta(P_i - \alpha) + \frac{1}{2}[P_i - P_j - \delta(P_i - \alpha) - \delta(P_j - \alpha)]$.

6. $\alpha > \beta > P_i > P_j$: The two agents will choose to trade with each other only if $P_i - P_j \geq \delta(\beta - P_i) + \delta(\beta - P_j)$ in which case Agent j gets $\delta(\beta - P_j) + \frac{1}{2}[P_i - P_j - \delta(\beta - P_i) - \delta(\beta - P_j)]$ and Agent i receives $\delta(\beta - P_i) + \frac{1}{2}[P_i - P_j - \delta(\beta - P_i) - \delta(\beta - P_j)]$, $i = 1, 2, j = 1, 2$, and $i \neq j$.

It is clear from above that

LEMMA. *The necessary and sufficient condition for the two parties to trade with each other is that $|P_i - P_j| > \delta[\max\{0, P_i - \alpha, \beta - P_i\} + \max\{0, P_j - \alpha, \beta - P_j\}]$.*

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